~~Antennae~~

Isotropic radiator | unipole: which radiates uniformally in all directions.

$W = P \cdot 4\pi r^2$ $\quad P =$ Poynting vector (avg. power density) (W/m^2)

$W =$ Total power radiated (W)

Hertzian Pole: Infinite current element $I\,dl$

I is uniform (const. through dipole) current $= I_0 \cos\omega t$

$$\boxed{P_{rad} = \frac{1}{2} I_0^2 R_{ad}} \rightarrow \text{for all antennae}$$

P_{rad}: time avg. radiated power

R_{ad}: Rad^n resist.

Assumption: $dl \leq \lambda/10$

$$\boxed{R_{rad} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2}$$

$$H_{\phi s} = \frac{j I_0 \beta dl}{4\pi r} \sin\theta \; e^{-j\beta r} \qquad E_{\theta s} = \eta H_{\phi s}$$

In fact, the fields vary a/c to r as

$1/r^3$ — electrostatic ~~dipole~~ field — very close to dipole

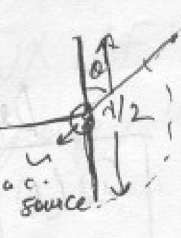
$1/r^2$ — inductive field — ~~close~~ near current ele.

$1/r$ — far / rad^n field — far from current ele. $(\beta r >> 1)$

Boundary b/w near and far zones made by $2d^2/\lambda$

$d =$ largest dimension of antenna.

~~Hert~~ Half-wave dipole



$$H_{\phi s} = \frac{j I_0 e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}, \quad E_{\theta s} = \eta H_{\phi s}$$

$R_{rad} \simeq 73\,\Omega$

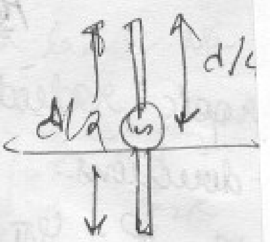
Total i/p Z of antenna, $Z_{in} = R_{in} + j X_{in}$

where $R_{in} \simeq R_{rad}$ (lossless antenna)

$X_{in} = 42.5\,\Omega$ $\quad l = \lambda/2$.

Resonance at $X_{in} = 0$, $\quad l = 0.485\lambda$

Quarter wave Monopole



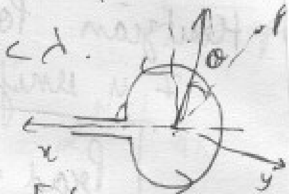
$H_{\phi s}$ same as half-wave dipole

$$R_{rad} \simeq 36.5\,\Omega$$

$$Z_{in} = 36.5 + j\,21.25\,\Omega$$

Small Loop antenna

Used as dir. finder in radio detector, T.V.$_2$ ante... at ultrahigh freq. "small" means $\rho_0 << \lambda$.



$$E_{\phi s} = \frac{120\pi^2 I_0 S}{r\,\lambda^2}\sin\theta \cdot e^{-j\beta r}$$

$$H_{\theta s} = -\frac{E_{\phi s}}{\eta}$$

$S = \pi\rho_0^2$ = loop area [ρ_0 = radius of loop]

$$R_{rad} = \frac{320\,\pi^4 S^2}{\lambda^4}$$

Radiation resist of dipole antenna of length l = $80\pi^2\left(\frac{l}{\lambda}\right)^2$

Parameters

Rad^n intensity $\boxed{U(\theta,\phi) = r^2 P_{ave}}$

$$P_{rad} = \oint_S P_{ave}\cdot dS = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\,d\Omega$$

where $d\Omega = \sin\theta\,d\theta\,d\phi$ — diff. solid angle in steradian (sr)

$$\boxed{U_{ave} = \frac{P_{rad}}{4\pi}}$$

Directive Gain: conc. of radiated power in a particular dir. θ,ϕ

$$\boxed{G_d(\theta,\phi) = \frac{U(\theta,\phi)}{U_{ave}}} = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

depends on θ,ϕ ← $\boxed{P_{ave} = \frac{G_d}{4\pi r^2}P_{rad}}$

Directivity (D) ratio of $\frac{\text{max. rad. intensity}}{\text{avg. rad. intensity}}$

$$D = \frac{U_{max}}{U_{ave}} = G_{d,max}$$

~~Some~~ D of some antennae

Isotropic	1
Hertzian dipole	1.5
" monopole	3
$\lambda/2$ dipole	1.64
$\lambda/4$ monopole	3.28

Power Gain (G_p) $\quad P_e$ = ohmic power loss

$$P_{in} = P_e + P_{rad}$$

$$G_p(\theta, \phi) = \frac{4\pi U(\theta,\phi)}{P_{in}}$$

Rad^n efficiency $\eta_r = \frac{G_p}{G_d} = \frac{P_{rad}}{P_{in}} = \frac{R_{ad}}{R_e + R_{rad}}$

Antenna Arrays

group of rad. elements arranged so as to produce some particular rad^m char.

Array Factor

$$AF = 2\cos\left[\frac{1}{2}(\beta d\cos\theta + \alpha)\right] \cdot e^{j\alpha/2}$$ for two element array

α = Phase diff. ~~of~~ current b/w the elements

d = dist. b/w them.

E(total) = E (due to single element at origin) X AF

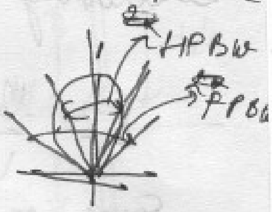
Resultant pattern = Unit pattern x Group pattern

In general N-element array

$$AF = \frac{\sin N\psi/2}{\sin \psi/2} \cdot e^{j\alpha/2} \qquad \psi = \beta d\cos\theta + \alpha$$

α → ~~consecutive~~ phase-diff. b/w consecutive elements.

HPBW Half-Power BW (Beam width) → Angle at which power becomes 1/2 or E becomes $1/\sqrt{2}$.



FNBW Angle at which $E = 0$.

Ω_A (Beam Area) $= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi$

Beam efficiency $= \dfrac{\Omega_M}{\Omega_A}$ Ω_M = main beam area

$$\boxed{D = \frac{4\pi}{\Omega_A}}$$

Resolution $\simeq$ FNBW/2

Directivity is equal to the no. of pt. sources in the sky that the antenna can resolve.

Aperture

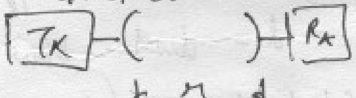
$\epsilon_{ap} = \dfrac{A_e}{A_p}$ A_p = physical aperture
A_e = effective " "
ϵ_{ap} = aperture efficiency

$$\boxed{A_e = \frac{\lambda^2}{\Omega_A} = \frac{\lambda^2 D}{4\pi}}$$

Friis X-mission formula

$P_r = G_{dr} G_{at} \left[\dfrac{\lambda}{4\pi r}\right]^2 P_t$

A_{et}, P_t, G_{dt} A_{er}, P_r, G_{dr}



G_d: directive gain $P_t = P_{rad}$ of Tx

Radar X-mission eq^n.

$P_r = \dfrac{(\lambda G_d)^2 \sigma P_{rad}}{(4\pi)^3 r^4}$ σ = scattering (radar) cross section of target