Digital X-mission

Quantisation Noise

$Q_N = \Delta^2/12$ PCM $\frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q_e^2 \, dq_e$

$Q_N = \Delta^2/3$ DM $\frac{1}{2\Delta}\int_{-\Delta}^{\Delta} q_e^2 \, dq_e$ $N_0 = \frac{\delta^2 B}{3 f_s}$

Condition for no slope overload

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

BW for TDM

$$f_{TDM} = \frac{1}{2T} = \frac{n}{2T_s} = \frac{n}{2} f_s \geq n f_m \text{ Hz.}$$

n = no. of i/p ~~signals~~ bits in complete cycle

$T_s = 1/f_s \leq 1/2f_m$

T = time b/w adjacent samples in time-mux signal

$$\boxed{f_{TDM} \geq n f_m}$$

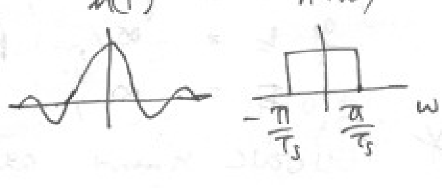
In case bit rate capacity of channel is given.

$$\boxed{R_b \geq n f_s}$$

ISI

a) Pulse Shaping

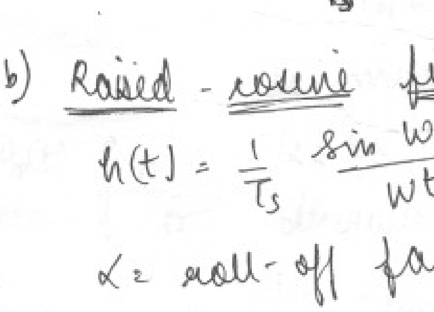
$$h(t) = \frac{1}{T_s} \frac{\sin \pi t/T_s}{\pi t/T_s}$$

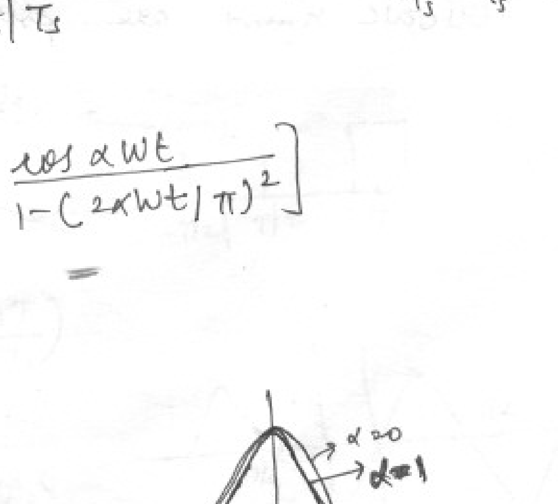


b) Raised-cosine filter

$$h(t) = \frac{1}{T_s} \frac{\sin Wt}{Wt}\left[\frac{\cos \alpha W t}{1-(2\alpha W t/\pi)^2}\right]$$

α = roll-off factor





$$H(\omega) = \begin{cases} 1 & 0 \le |\omega| \le (1-\alpha)W \\ \frac{1}{2}\left\{1 - \sin\left[\frac{\pi}{2\alpha W}(|\omega| - W)\right]\right\} & (1-\alpha)W \le |\omega| \le (1+\alpha)W \\ 0 & |\omega| > (1+\alpha)W \end{cases}$$

$W = \pi T_s$

$$f_B = \frac{1+\alpha}{2T_s} \text{ Hz}$$

Energy & Power Signals.

For discrete

$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$ If finite it is energy signal

if $E = \infty$

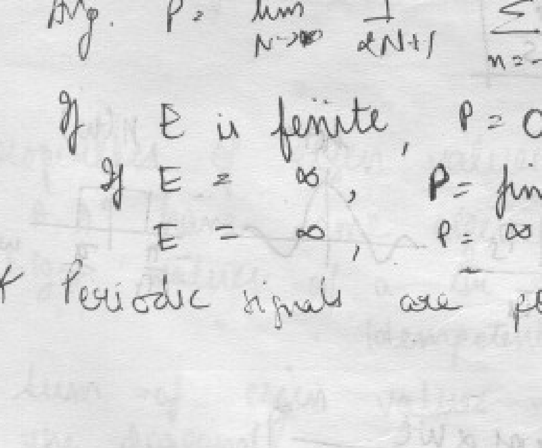
Avg. $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

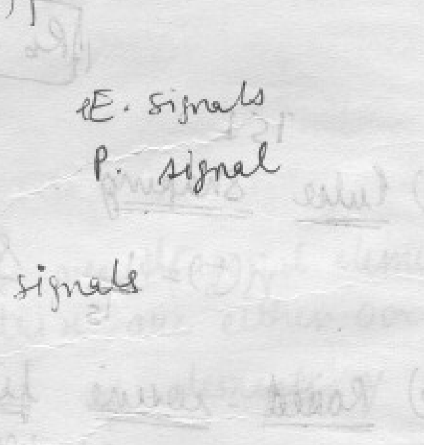
If E is finite, $P = 0$ — E. signals

If $E = \infty$, $P =$ finite — P. signal

$E = \infty$, $P = \infty$

* Periodic signals are power signals





Fourier Analysis

Dirichlet's conditions

1. $g_p(t)$ is single-valued
2. finite no. of discontinuities, maxima / minima in one period
3. $g_p(t)$ is abs. integrable

$$\int_{-T_0/2}^{T_0/2} |g_p(t)| \, dt < \infty$$

$$g_p(t) = a_0 + 2\sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi n t}{T_0}\right) + b_n \sin\left(\frac{2\pi n t}{T_0}\right)\right].$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t)\, dt$$

$$a_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t) \cos\left(\frac{2\pi n t}{T_0}\right) \cdot dt$$

$$b_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t) \sin\left(\frac{2\pi n t}{T_0}\right) \cdot dt$$

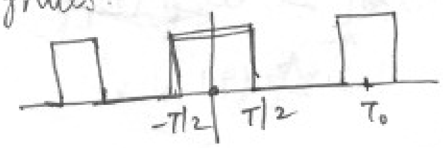
Complex Fourier series

$$g_p(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{(j2\pi n t/T_0)}$$

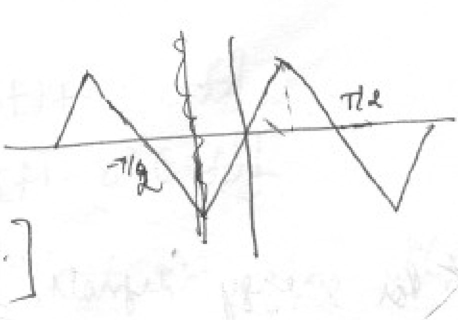
$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p(t) \cdot e^{(-j2\pi n t/T_0)} \cdot dt \,, \quad n = 0, \pm 1, \ldots$$

Some common signals

$$g_p(t) \begin{cases} A & -T/2 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases}$$



$$c_n = \frac{TA}{T_0} \operatorname{sinc}\left(\frac{nT}{T_0}\right)$$



$$g_p(t) = \frac{8A}{\pi^2}\left[\sin \omega_0 t - \frac{1}{3^2}\sin 3\omega_0 t + \frac{1}{5^2}\sin 5\omega_0 t - \cdots\right]$$

Some prop.

(i) If $f(-t) = f(t)$ (EVEN) only cos terms

(ii) If $f(-t) = -f(t)$ (ODD) only sin terms

(iii) If $f(t+T/2) = f(t)$ only even harmonics — mirror of $f(t)$ w.r.t $T/2$ equals $f(t)$

(iv) If $f(t+T/2) = -f(t)$ only odd harmonics, waveform has half-wave symmetry

Response of LTI sys. to $u(t) \rightarrow a(t)$

$$a(t) = \int_{-\infty}^{t} h(\tau)\cdot d\tau$$

eigenvalue & eigen~~vector~~ func. ~~of~~ LTI sys.

~~Operator~~ τ = LTI sys. with impulse response $h(t)$

If $\tau[x(t)] = \lambda\, x(t)$

λ = eigenvalue $= H(\omega)$

$x(t)$ = eigen ~~vector~~ func^n $= e^{j\omega t}$

~~Fourier~~ ~~X form~~

~~Transition~~ Time		Freq.
Continuous aperiodic	$\longrightarrow$	Continuous aperiodic
continuous periodic	$\longrightarrow$	Discrete aperiodic
Discrete aperiodic	$\longrightarrow$	Continuous periodic
Discrete ~~ap~~ periodic	$\Longrightarrow$	~~Discrete~~ Discrete. periodic

★ For energy signals $\int_{-\infty}^{\infty} |g(t)|^2\, dt < \infty$

Fourier X-form

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-(j2\pi ft)} \, dt$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{j2\pi ft} \, dt$$

* For real-valued func $g(t)$ $|G(-f)| = |G(f)|$ and $\theta(-f) = -\theta(f)$] conjugate symm.

Prop.

* Linearity $a g_1(t) + b g_2(t) \rightleftharpoons a G_1(f) + b G_2(f)$
* Time Scaling $g(at) \rightleftharpoons \frac{1}{|a|} G\left(\frac{f}{a}\right)$
* Duality If $g(t) \rightleftharpoons G(f)$ then $G(t) \rightleftharpoons g(-f)$
* Time shifting $g(t - t_0) \rightleftharpoons G(f) \cdot e^{-j2\pi f t_0}$
* Freq. shifting $e^{j2\pi f_c t} g(t) \rightleftharpoons G(f - f_c)$
* Diff. in time domain $\frac{d^n}{dt^n} g(t) \rightleftharpoons (j2\pi f)^n G(f)$
* Gaussian pulse is its own Fourier X-form $e^{(-\pi t^2)} \rightleftharpoons e^{(-\pi f^2)}$
* Integration in time domain $\int_{-\infty}^{t} g(t) \cdot dt \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{1}{2} G(0) \delta(f)$
* Conjugate func $g^*(t) \rightleftharpoons G^*(-f)$
* Multiplication in time $g_1(t) \cdot g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda) G_2(t - \lambda) \, d\lambda$ $= G_1(f) \star G_2(f)$

Convolution in time $\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau) \, d\tau \rightleftharpoons G_1(f) \cdot G_2(f)$

$$\mathrm{Re}[g(t)] \rightleftharpoons \frac{1}{2}[G(f) + G^*(-f)]$$

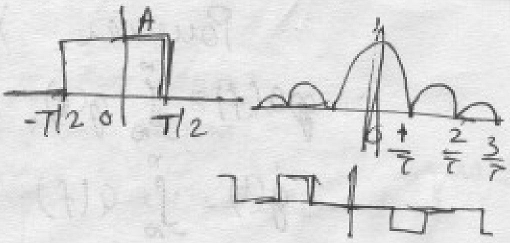
$$\mathrm{Im}[g(t)] \rightleftharpoons \frac{1}{2j}[G(f) - G^*(-f)]$$

Complex conjugation for real-valued signal $g(-t) \rightleftharpoons G^*(f)$

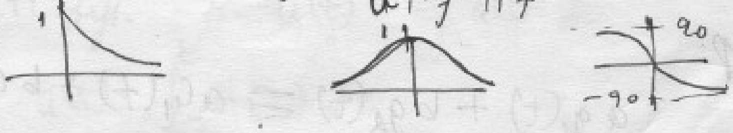
Some common X-forms

* $g(t) = A \,\mathrm{rect}(t/T)$

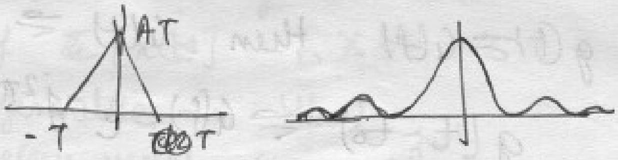
$\rightleftharpoons AT \,\mathrm{sinc}(fT)$



* $e^{-at} u(t) \rightleftharpoons \dfrac{1}{a + j2\pi f}$



* $A \,\Delta(t/T) \rightleftharpoons AT^2 \,\mathrm{sinc}^2(fT)$



* $\delta(t) \rightleftharpoons 1$

$g(t) \star \delta(t) = g(t)$

$\int_{-\infty}^{\infty} g(t) \cdot \delta(t - t_0) = g(t_0)$

* $\cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$

* $\sin(2\pi f_c t) \rightleftharpoons \frac{1}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]$

* $\mathrm{sgn}(t) \rightleftharpoons \dfrac{1}{j\pi f}$

$u(t) \rightleftharpoons \dfrac{1}{j\pi f} + \dfrac{1}{2}\delta(f)$

Differentiation

$y = f(x)$	$D^n y$
$(ax+b)^m$	$m(m-1)\ldots(m-\overline{n-1})\, a^m (ax+b)^{m-n}$
$y = \dfrac{1}{(ax+b)}$	$\dfrac{(-1)^n \lfloor n \cdot a^n}{(ax+b)^{n+1}}$
$y = \log(ax+b)$	$\dfrac{(-1)^{n-1} \lfloor n-1 \; a^n}{(ax+b)^n}$
$\sin(ax+b)$	$a^n \sin(ax+b+n\pi/2)$
$\cos(ax+b)$	$a^n \cos(ax+b+n\pi/2)$
$e^{ax} \cdot \sin(bx+c)$	$(a^2+b^2)^{n/2} \cdot e^{ax}[\sin(bx+c+n\tan^{-1} b/a)]$
$e^{ax} \cdot \cos(bx+c)$	$(a^2+b^2)^{n/2} \cdot e^{ax}[\cos(bx+c+n\tan^{-1} b/a)]$
a^{mx}	$(m \log a)^n \cdot a^{mx}$
e^{mx}	$m^n \cdot e^{mx}$

Leibnitz Theorem

$$(uv)_n = u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \ldots + {}^nC_n u v_n$$

Cauchy's Mean Value Theorem

If (i) $f(x)$, $g(x)$ be continuous in $[a, b]$

(ii) $f'(x)$, $g'(x)$ exist in (a, b)

(iii) $g'(x) \neq 0$ for any value of x in (a, b)

then there exists $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

If $g(x) = x$, gets converted to Lagrange's M.V. theorem